

class - XII (Sci) Home-work for Summer Vacation 2017

Sub: Mathematics

17. Determine whether each of the following relations are reflexive, symmetric and transitive:

a) Relation  $R$  in the set  $A = \{1, 2, 3, \dots, 13, 14\}$  defined as

$$R = \{(x, y) : 3x - y = 0\}$$

b) Relation  $R$  in the set  $N$  of natural numbers defined as

$$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$$

2) Check the injectivity and surjectivity of the following functions:-

a)  $f: N \rightarrow N$  given by  $f(x) = x^2$

b)  $f: Z \rightarrow Z$  given by  $f(x) = x^3$

3) Let  $f: N \rightarrow N$  be defined by  $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$  for all  $n \in N$ .

State whether the function  $f$  is bijective. Justify your answer.

4) Let  $A = R - \{3\}$  and  $B = R - \{1\}$ . Consider the function  $f: A \rightarrow B$  defined

by  $f(x) = \left(\frac{x-2}{x-3}\right)$ . Is  $f$  one-one and onto? Justify your answer.

5) Let  $f: N \rightarrow R$  be a function defined as  $f(x) = 4x^2 + 12x + 15$ .

Show that  $f: N \rightarrow S$ , where  $S$  is the range of  $f$ , is invertible. Find the inverse of  $f$ .

6) Show that  $f: [-1, 1] \rightarrow R$ , given by  $f(x) = \frac{x}{(x+2)}$  is one-one.

Find the inverse of the function  $f: [-1, 1] \rightarrow \text{Range } f$ .

7). Consider  $f: \mathbb{R}_+ \rightarrow [4, \infty)$  given by  $f(x) = x^2 + 4$ . show that  $f$  is invertible with the inverse  $f^{-1}$  of  $f$  given by  $f^{-1}(y) = \sqrt{y-4}$ , where  $\mathbb{R}_+$  is the set of all non-negative real numbers.

8). Consider  $f: \mathbb{R}_+ \rightarrow [-5, \infty)$  given by  $f(x) = 9x^2 + 6x - 5$ . show that  $f$  is invertible with  $f^{-1}(y) = \left( \frac{\sqrt{y+6} - 1}{3} \right)$ .

9). If  $f: \mathbb{R} \rightarrow \mathbb{R}$  be given by  $f(x) = (3-x^3)^{1/3}$ , then  $f \circ f(x)$  is  
(A)  $x^{1/3}$  (B)  $x^3$  (C)  $x$  (D)  $(3-x^3)$ .

10). Miscellaneous Exercise on Chapter 1.

11). Exercise 2.2.

12). Miscellaneous Exercise on Chapter 2.

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